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Application of the David Taylor Navier-Stokes (DTNS) Code in Non-Inertial Reference Frames

by

Joseph J. Gorski

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CONTENTS

Page

FIGURES

ABSTRACT

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INTRODUCTION

There has been a tremendous effort in recent years to develop Navier-Stokes flow solvers for computing complex flow fields. The results provide numerical pictures of flow fields by which complex fluid dynamics phenomena can be investigated in more detail than typically available with experiments. As these Navier-Stokes flow solvers become more reliable they can be used to impact the design process as demonstrated in Refs. [1] and [2].

A primary motivation for this work is to investigate the use of Reynolds averaged Navier-Stokes flow solvers for studying the effect of propulsors on the stability performance of a submersible vehicle in turn. Although this is in reality an unsteady flow phenomenon it can be modelled as a body rotating about an axis and solved with the steady state Navier-Stokes equations in a non-inertial reference frame. Another area of interest is the flow through turbomachinery and propulsors. These flows are particularly complex because of the tip vortex formation, rotation of the flow field, and interaction of blade and hub boundary layers. Current theories used in the design process are based on potential flow assumptions, that are not strictly applicable for the complexity of the flow, or experimental data bases. Simple body force models added to Navier-Stokes flow solvers, Refs. [2-4], have done an adequate job of modelling the gross features of turbomachiniery flow fields but do not provide a detailed description of the flow and are of limited use because of the basic reliance on potential theories. A full Navier-Stokes solution capability for such flow fields can have a considerable impact on the design and understanding of this complex flow phenomena. Although the flow is unsteady in the absolute frame it is often steady in the rotating frame. By augmenting the David Taylor Navier-Stokes (DTNS) flow code to include non-inertial reference frames a tool will be available to begin investigating complex flows with rotation.

The DTNS Reynolds-averaged Navier-Stokes (RANS) flow solvers were developed under the Numerical Analysis of Naval Fluid Dynamics Accelerated Research Initiative sponsored by the Office of Naval Research. The purpose was the development of methods to facilitate the analysis of high Reynolds number flows in naval geometries in which viscous effects can neither be neglected nor modelled by approximate formulations. The codes, including two-dimensional (DTNS2D), axisymmetric (DTNSA), and three-dimensional (DTNS3D) versions, were designed to be relatively easy to use for computational fluid dynamics (CFD) analysis. These codes can be applied to a wide variety of internal and external flow fields as demonstrated in Refs. [1,2,5-10].

An important aspect of this work is an investigation of which presently available turbulence models are adequate for the calculation of flows in non-inertial reference frames. The original DTNS computer codes contain the Baldwin-Lomax [11] and standard $k-\epsilon$ [12] models but, as shown by Speziale [13], these models are incapable of accurately predicting rotational effects. Speziale also points out inconsistencies with the various modifications that others have made to the two-equation models but with the unavailability of anything better these will be tested for the flow in a rotating channel. The only other recourse would be to include a second-order closure scheme such as that of So and Peskin [14], Mellor and Yamada [15], or Launder et al [16] but it is doubtful that these models, which require the solution of six additional partial differential equations, would be generally applicable in three-dimensional flow fields for which the DTNS computer codes were developed. Additionally, these more complex models have their own problems when computing rotating flows as demonstrated by Speziale [17]. The scope of this work will be limited to conventional eddy viscosity and algebraic Reynolds stress models.

NAVIER-STOKES EQUATIONS

The Reynolds averaged Navier-Stokes equations for steady incompressible flow in a non-inertial reference frame can be written as (c.f., Batchelor [18])

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{x_i} u_i u_j = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j}\tau_{ij} - e_{ipq} e_{qjk} \Omega_p \Omega_j X_k - 2 e_{ijk} \Omega_j u_k \tag{1}$$

This is the conservative form of the equations written in cartesian tensor notation. Here Ω_i is the rotation rate of the non-inertial reference frame relative to the inertial frame, X_i is the position vector, and e_{ijk} is the alternating unit tensor. The Reynolds stresses, τ_{ij}, need to be modelled for turbulent flow and are zero for laminar flow. The Navier-Stokes equations for a non-inertial reference frame take the same form as the equations for an inertial frame except for the last two terms on the right hand side of Eq. (1) which are the centrifugal and Coriolis accelerations respectively. It should be noted that Eulerian and translational accelerations are not included in the above equation because of the assumption of a steady state flow field. The local velocities in the non-inertial reference frame, u_i in Eq. (1), can be related to the absolute velocities in the inertial frame, U_i, using the following relation

$$U_i = u_i + e_{ijk} \Omega_j X_k \tag{2}$$

The flow in a non-inertial reference frame is also governed by the continuity equation which for an incompressible flow is

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{3}$$

The continuity equation is frame invariant and so takes the same form for both inertial and non-inertial reference frames. Because the above equations (1 and 3) have the same basic form in both inertial and non-inertial reference frames the DTNS codes can be directly applied in a non-inertial reference frame. The rotation terms are treated as source terms and do not affect the basic equation solver. It must be remembered that the code will now solve for the flow field relative to the rotating coordinate system.

Solution Procedure

Only a brief description of the solution procedure is given here as details can be found elsewhere [5,6]. The Navier-Stokes equations contain both first derivative convective terms and second derivative viscous terms. The viscous terms are numerically well-behaved diffusion terms

and are discretized using standard central differences. The upwind differenced Total Variational Diminishing (TVD) scheme developed by Chakravarthy et. al.[19] was used for discretizing the convective part of the equations. The Jacobian matrices of the convective terms are used to generate eigenvectors and eigenvalues for the system of equations. The convective terms are then forward or backward differenced based on the sign of the eigenvalues. This produces a third order accurate numerical scheme without any artificial dissipation terms being added to the equations for stability. The equations are transformed to a body fitted coordinate system and solved using a finite volume procedure.

The artificial compressibility technique of Chorin [20] is used to add a time derivative term for pressure to the continuity equation. This allows the system of equations to be marched in time in an implicit coupled manner using approximate factorization. The implicit side of the equations are discretized with a first-order accurate upwind scheme for the convective terms. This creates a diagonally dominant system which requires the inversion of block tri-diagonal matrices. The implicit side of the equations are only first order accurate but the final converged solution has the high order of accuracy of the explicit part of the equations.

Laminar Flow Calculation

To test the DTNS code in a non-inertial reference frame, the laminar flow between two concentric cylinders was computed with the inner cylinder rotating and the outer cylinder at rest, Fig. 1.

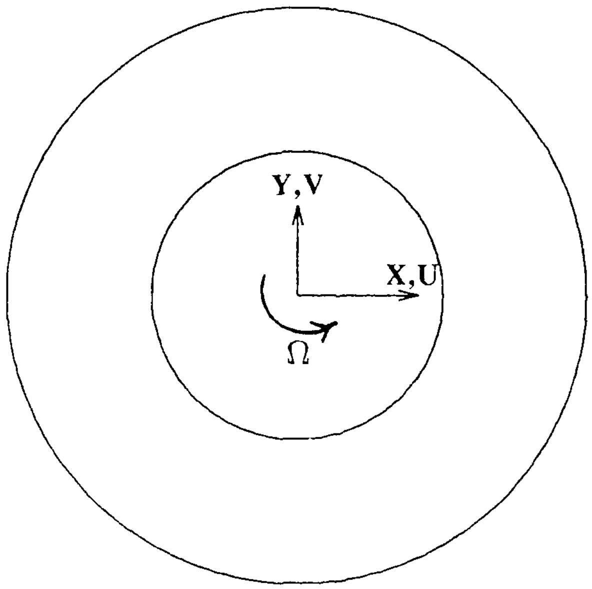

Fig. 1. Concentric cylinder problem.

This is analogous to a turbomachinery flow field with a hub rotating within an annulus without the complexity of blades. For this flow an exact solution can be obtained for the tangential velocity from the Navier-Stokes equations in polar coordinates (c.f., Schlichting [21]) since the radial velocity and all derivatives in the tangential direction are zero. However, this is a good test case for RANS codes, such as DTNS, which are written in cartesian coordinates as none of the velocities or derivatives can be eliminated and all the convective and viscous terms are tested. Thus, the X and Y velocity components would be different at each tangential location but when

they are combined to form a tangential velocity component it should be the same on each tangential plane.

The typical way to compute this flow is to generate a grid for the flow field and obtain the solution in an inertial frame. The tangential velocity on the inner cylinder would be set to some value and the tangential velocity on the outer cylinder would be set to zero. Note that for the DTNS codes these tangential velocities at the boundaries would need to be converted to appropriate velocities in the X and Y directions. To solve the problem in a non-inertial frame the same grid can be used but now it is assumed that the grid has an angular velocity Ω in the Z direction where $R\Omega = U_\theta$ on the inner cylinder. In the non-inertial frame the boundary condition on the inner surface is now no-slip with u and v set to zero. The outer cylinder is not moving in the absolute frame and hence the absolute velocities U and V must be zero there. With this information and Eq. (2) we can get the following boundary conditions for the relative velocities on the outer cylinder

$$u = \Omega y \quad \text{and} \quad v = -\Omega x.$$

The flow field was computed with an outer cylinder having twice the radius of the inner cylinder. This flow is independent of Reynolds number, $R_e(\equiv U_\theta R/\nu)$, and was computed with a Rossby number, $R_o(\equiv \Omega R/U_\theta)$, of 1. The computed relative velocity in the tangential direction is shown in Fig. 2.

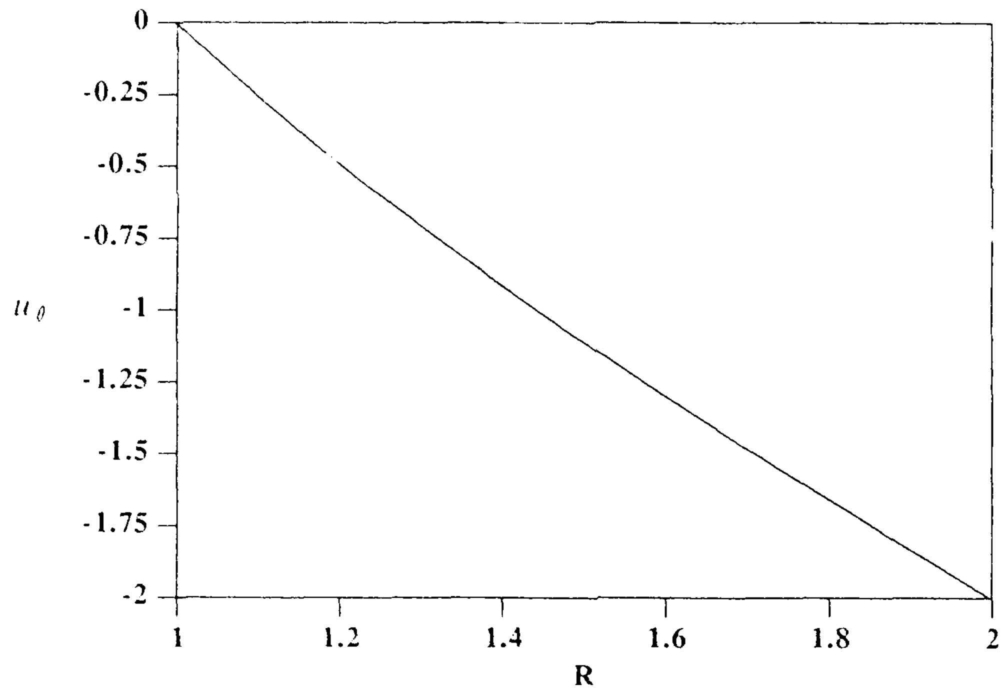

Fig. 2. Relative velocity for the concentric cylinders.

As can be seen it goes from 0 on the inner cylinder to -2 on the outer cylinder. When this is converted to the absolute frame using Eq. (2) excellent agreement with the exact solution is obtained as shown in Fig. 3.

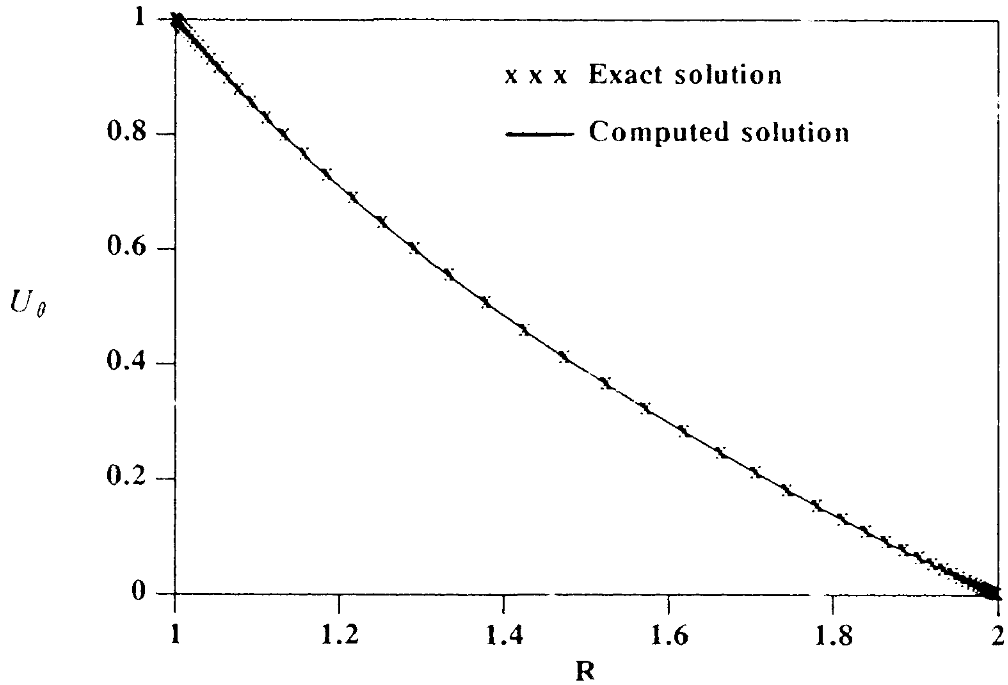

Fig. 3. Absolute velocity for the concentric cylinders.

TURBULENCE MODELLING

The calculation of laminar flow in a non-inertial reference frame poses no great difficulty with an existing Navier-Stokes flow solver. In order to compute turbulent flow the Reynolds stress terms in Eq. (1) must be computed using an appropriate turbulence model. Full Reynolds stress closure models have produced promising results for certain rotating flow fields (ie. Launder et al. [22]) but it is doubtful that these models would be generally applicable in three-dimensional flow fields for which the DTNS computer codes were designed. The added computational time of six extra partial differential equations for the stresses, with questionable degrees of accuracy, is also undesirable. A practical alternative is to model the Reynolds stresses using the Boussinesq eddy viscosity assumption

$$\tau_{ij} = \frac{2}{3}k\,\delta_{ij} - 2\nu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

Where k is the turbulent kinetic energy, δ_{ij} is the Kronecker delta, and ν_t is the eddy viscosity. The kinetic energy can be absorbed into the pressure term of the Navier-Stokes equations but it is necessary to compute the eddy viscosity using an appropriate turbulence model. A variety of turbulence models have been proposed for computing rotating flows within the eddy viscosity framework, some of which are discussed by Lakshminarayana [23]. Speziale [13] has shown theoretically that most of these models are not strictly applicable in rotating flows but since nothing better is currently available some of these models have been evaluated here.

A variety of models have been tested for the flow in a rotating channel as measured by Johnston et al. [24], Fig. 4. This is a fully developed two-dimensional channel flow for which centrifugal effects are negligible and the Coriolis forces dominate the rotating flow field. The cases considered are for a Reynolds number of 11500 with Rossby numbers of 0.069 and 0.210 and a Reynolds number of 35000 with a Rossby number of 0.068. These Reynolds and Rossby numbers

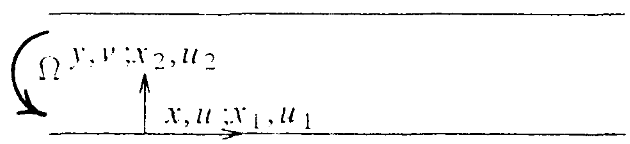

Fig. 4. Rotating channel problem.

are based on the bulk mean velocity within the channel and the channel diameter. The nonrotating flow is symmetric about the channel centerline and can be computed quite well with existing algebraic and $k-\epsilon$ models, Fig. 5.

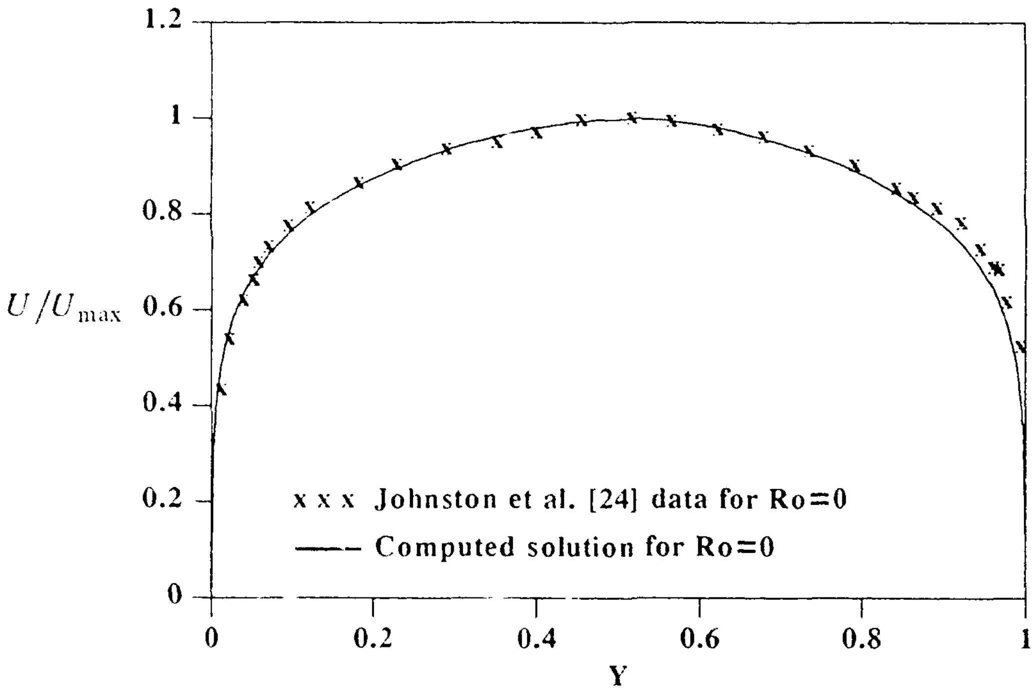

Fig. 5. Computed profile for Ro=0.

The laminar flow for the rotating cases is also symmetric about the channel centerline because of the negligible centrifugal forces. The turbulent flow is nonsymmetric about the channel centerline and the asymmetry in the problem is due to the Reynolds stresses. Therefore, even though the flow itself is relatively simple, it is necessary to model the correct turbulent flow behaviour to compute the flow accurately. As stated by Launder et al. [22] eddy viscosity models developed for inertial reference frames will produce a symmetric profile for this problem. Evidence of this can be seen in Fig. 6 for the Baldwin-Lomax model at a Rossby number of 0.21. The profile is

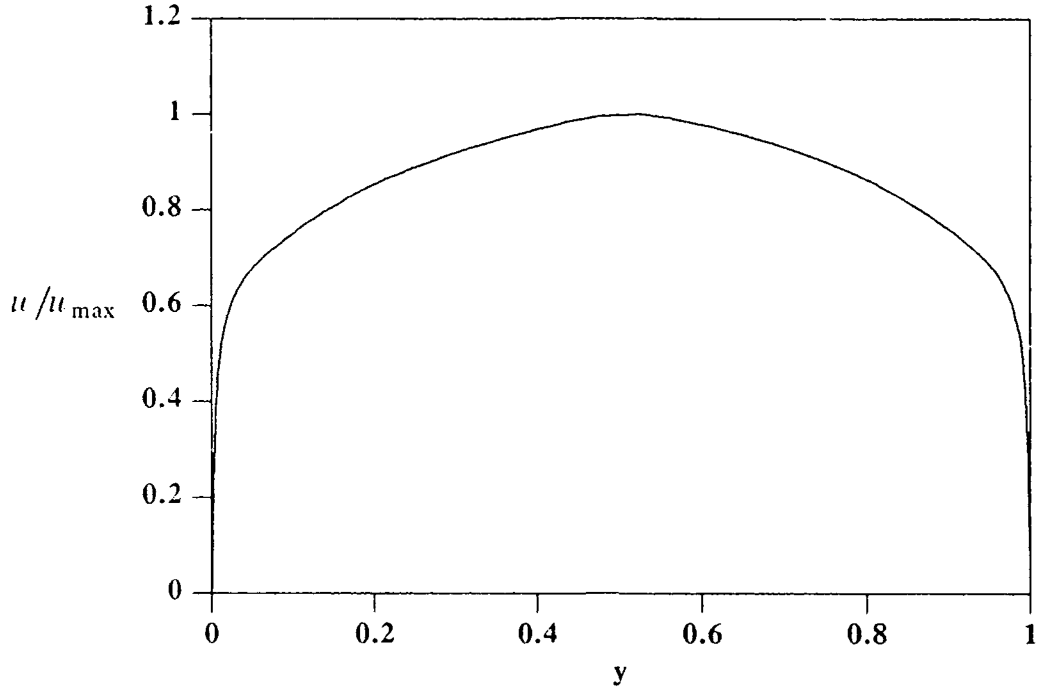

Fig. 6. Computed profile with the Baldwin-Lomax model for Ro=0.21

symmetric about the channel centerline and does not exhibit any effects of rotation on the flow field. Because of this shortcoming of algebraic eddy viscosity models most of the reported attempts at predicting rotating turbulent flow have relied on the $k-\epsilon$ model.

$k-\epsilon$ Model

The modelled form of the $k-\epsilon$ equations for an inertial reference frame can be written as

$$\frac{\partial k}{\partial t} + \frac{\partial k u_i}{\partial x_i} = \frac{\partial}{\partial x_i}(\nu + \nu_t)\frac{\partial k}{\partial x_i} - \tau_{ij}\frac{\partial u_i}{\partial x_j} - \epsilon \tag{5}$$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial \epsilon u_i}{\partial x_i} = \frac{\partial}{\partial x_i}(\nu + \nu_t/\sigma)\frac{\partial \epsilon}{\partial x_i} - C_1\frac{\epsilon}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j} - C_2\frac{\epsilon^2}{k}$$

The values of k and ϵ are then used to compute an eddy viscosity using

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \tag{6}$$

Typical constants for the model are $C_1 = 1.44$, $C_2 = 1.92$, and $C_\mu = 0.09$. The $k-\epsilon$ equations have the same form as the Navier-Stokes equations with convective and diffusion terms and hence can be solved with the same discretization technique. Details of the upwind differenced scheme as applied to the $k-\epsilon$ equations can be found in Ref. [6]. All of the models studied here have been previously applied using the law of the wall boundary conditions of Launder and Spalding [12] or the slip velocity of Kreskovsky et al [25], which is essentially equivalent. However, these boundary conditions can produce substantial errors for complex three-dimensional flows as alluded to by Gorski et al. [26]. Therefore, the present computations bridged the viscous sublayer using the near-wall formulation of Gorski [27]. The $k-\epsilon$ equations are invariant under the

transformation to a non-inertial reference frame and hence the equations (5) are the same whether or not the system is rotating. As shown by Raj [28] there is a rotation term in the ϵ equation but this vanishes with the assumption of isotropic turbulence. Hence, the standard $k-\epsilon$ model will also produce a symmetric profile about the channel centerline for a Rossby number of 0.21 as shown in Fig. 7.

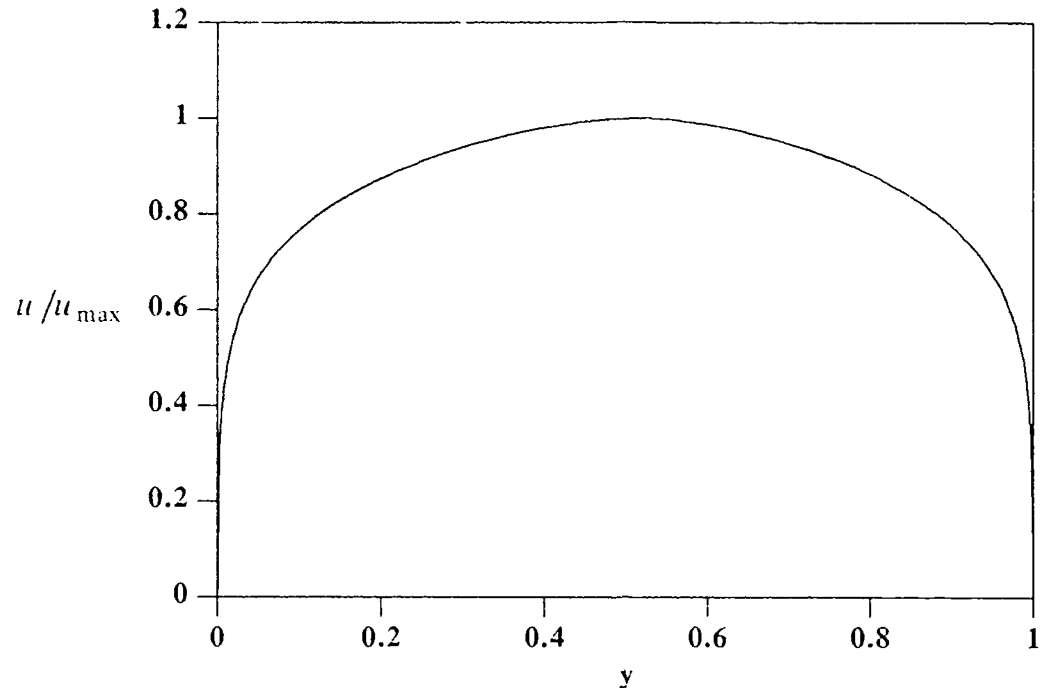

Fig. 7. Computed profile with the $k-\epsilon$ model for Ro=0.21

Because the channel flow is fully developed at the experimental measuring station it is fairly straightforward to examine why the $k-\epsilon$ model produces a symmetric profile. For this flow all derivatives are zero except $\partial/\partial y$. The v velocity component is also zero. To produce an asymmetric flow field the equations need an asymmetric term. The term $\partial u/\partial y$ is asymmetric being positive below the centerline and negative above the centerline but the derivative of this with respect to y as it appears in the Navier-Stokes equations is symmetric. Similarly the Reynolds stress term τ_{12} changes sign across the channel centerline but its derivative is symmetric. This is why the flow is symmetric without rotation. For the rotation to produce an asymmetric flow field there must be an asymmetric term linked to the angular velocity, Ω. The centrifugal force term in the Navier-Stokes equations is not symmetric across the channel centerline but this term is negligible for this flow. The Coriolis force term is symmetric across the channel centerline and cannot produce an asymmetric flow field. Therefore the asymmetric rotational term must be contained in the turbulence model. There is no asymmetric term in the standard $k-\epsilon$ model and hence it cannot produce an asymmetric flow. Several models have been proposed to account for rotation in the turbulence model most of which have been investigated in the present work.

Modified $k-\epsilon$ Models

Various models, two of which are investigated here, have included rotation effects directly in the k and ϵ equations. Wilcox and Chambers [29] modified the equations by adding

$$9\nu_t \frac{\partial u}{\partial y}$$

to the kinetic energy equation and

$$9\nu_t (\frac{\epsilon}{k}) \frac{\partial u}{\partial y}$$

to the dissipation equation. A similar approach was taken by Howard et al. [30] who did not modify the kinetic energy equation but replaced the constant C_2 in the dissipation equation by a turbulent Richardson number of the form

$$C_2(1 + 0.4(\frac{k}{\epsilon})^2 \Omega \frac{\partial u}{\partial y})$$

Since no such terms appear explicitly in the exact kinetic energy equation these techniques appear to be quite ad hoc. However, it can be argued that adding rotation terms to the dissipation equation is an attempt to model the anisotropic rotation terms that appear in the exact form of the equation. The results obtained with the two models are nearly identical as shown in Figs. 8-10.

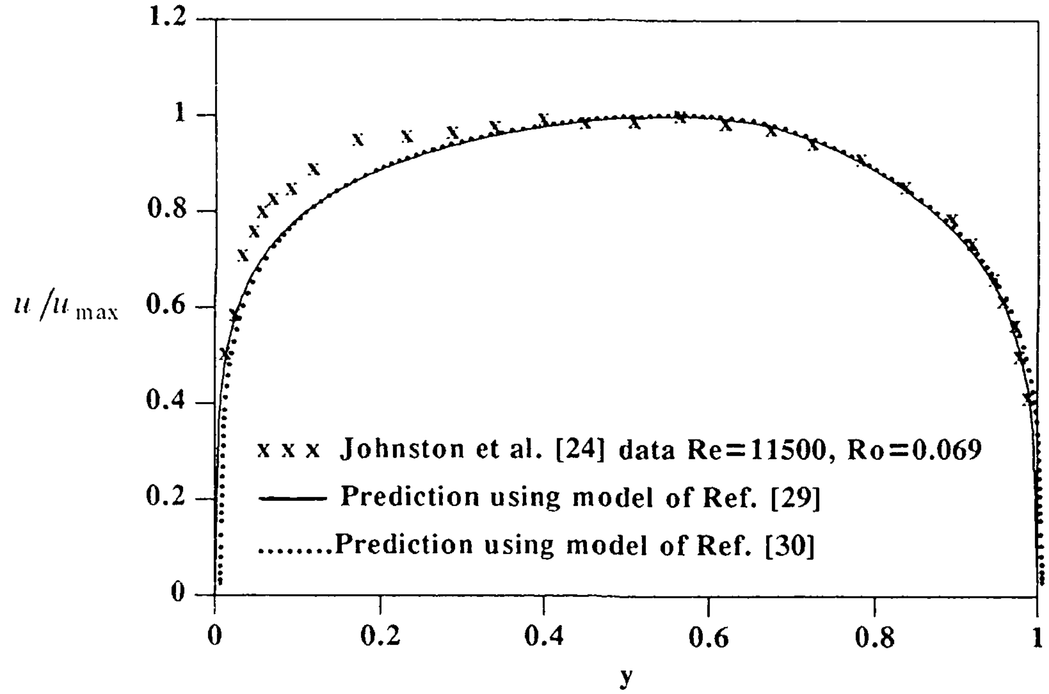

Fig. 8. Computed profiles with the modified $k-\epsilon$ models for Re=11500, Ro=0.069.

The models do produce asymmetry in the flow because of the $\partial u/\partial y$ term. Because this term changes sign across the channel it enhances production on the unstable side of the channel, below the centerline where $\partial u/\partial y$ is positive (y = 0), and decreases production on the stable side of the channel where $\partial u/\partial y$ is negative. Both models underpredict the velocity on the unstable side of the channel for all Rossby numbers. Although the predictions right next to the wall are quite good the predictions do not compute the steep rise in velocity evident in the experiment. This seems to indicate that the near-wall model used is adequate and the discrepancies on the unstable side are more a result of the rotation effects in the $k-\epsilon$ models. For the low Rossby number cases the predictions on the stable side of the channel are very good out past the symmetry plane. However,

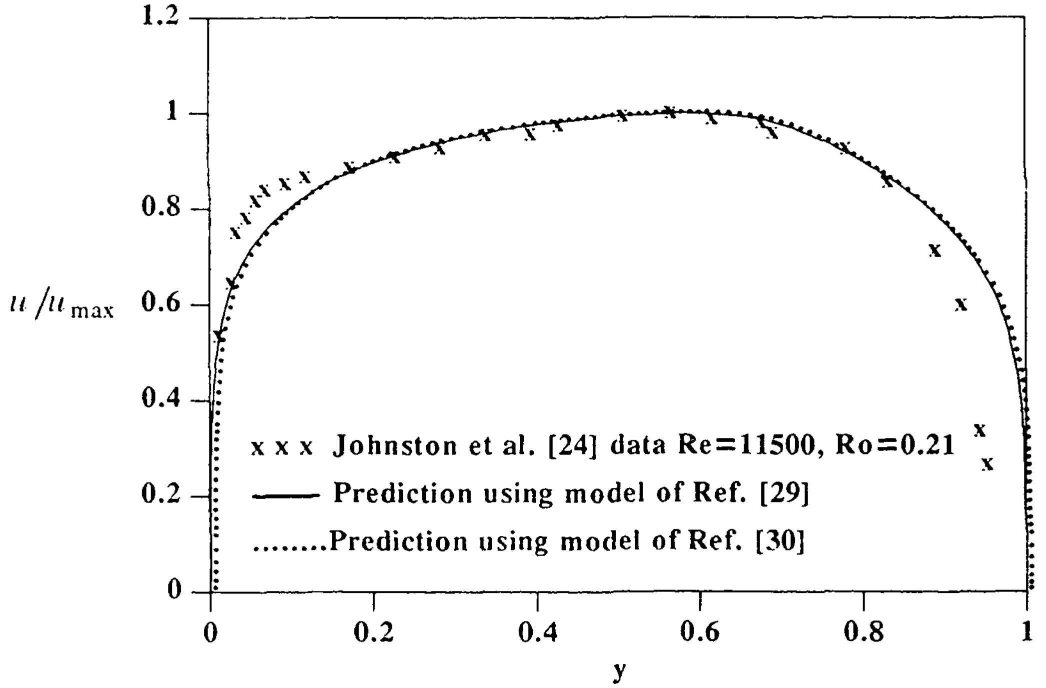

Fig. 9. Computed profiles with the modified $k-\epsilon$ models for Re=11500, Ro=0.21.

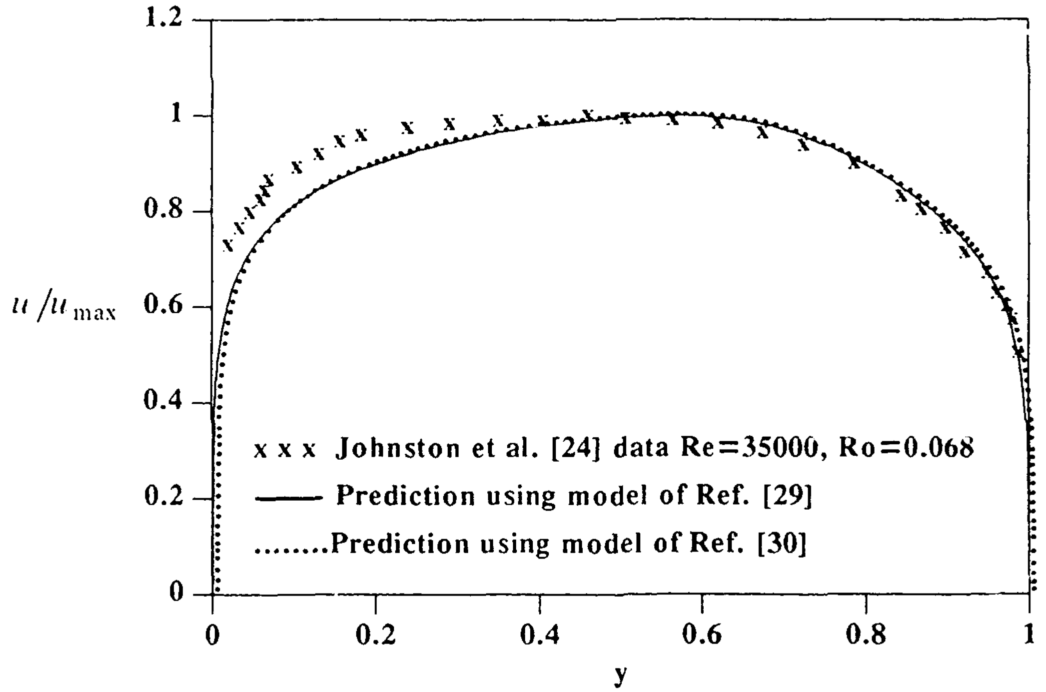

Fig. 10. Computed profiles with the modified $k-\epsilon$ models for Re=35000, Ro=0.068.

there are considerable differences between the predicted and experimental values near the wall for the high Rossby number case (Ro=0.21). This can be due to problems with the modelled rotation terms at high rotation rates or to improperly accounting for high rotation rates in the near-wall region. The experiments also indicate there is a significant laminar region on the stable side of the channel for this high Rossby number case and use of $k-\epsilon$ turbulence models may be inadequate here in general.

A recent investigation by Bardina [31] produced a modified $k-\epsilon$ model for rotation based on full simulation data. The constants C_1 and C_2 in the dissipation equation were modified by a term of the form

$$\pm C_\Omega \frac{\Omega k}{\epsilon}$$

This term is symmetric across the channel and hence cannot produce the asymmetric profile of the experiment.

Modified C_μ Models

An alternative approach to modifying the $k-\epsilon$ equations is to solve the standard $k-\epsilon$ equations but modify the computation for eddy viscosity, Eq. (6). This method typically starts with the full Reynolds stress equations which are then simplified to produce an algebraic model for the Reynolds stresses as demonstrated by Galmes and Lakshminarayana [32]. This algebraic model is then futher simplified to obtain a relation for the eddy viscosity based on the angular velocity. The standard Boussinesq eddy viscosity assumption is still used for the Reynolds stresses so only the eddy viscosity is modified from the original equations. This modification is introduced into the eddy viscosity computation by changing the constant C_μ. One such variation is that of Pouagare and Lakshminarayana [33] where

$$C_\mu = 0.09 + \frac{\Omega}{\partial u/\partial y}$$

which reduces to the standard definition of C_μ for no rotation. A more involved model was developed by Warfield and Lakshminarayana [34] which takes the form

$$C_\mu = -\frac{2}{3}(C_3 - 1)(C_3\frac{P}{\epsilon} + C_4 - 1)/(D_1 + D_2)$$

where

$$D_1 = (\frac{P}{\epsilon})^2 + 2\frac{P}{\epsilon}(C_4 - 1) + (C_4 - 1)^2$$

$$D_2 = [2R_1(2 - C_{3)}]^2 + 4(C_3 - 1)(2 - C_3)R_1{}^2R_2$$

with

$$R_1 = \frac{k\Omega}{\epsilon} \quad \text{and} \quad R_2 = \frac{\partial u/\partial y}{\Omega}.$$

P is the production of kinetic energy given by

$$P = -\tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{7}$$

The constants C_3 and C_4 were set to 0.6 and 1.5 respectively. Because of the change in sign of $\partial u/\partial y$ across the centerline these models produce an asymmetric profile by increasing C_μ, and

hence eddy viscosity, on the unstable side and decreasing it on the stable side of the channel. Care must be taken when using these models as C_μ will increase or decrease depending on the sign of $\partial u/\partial y$. Near the centerline $\partial u/\partial y$ approaches zero creating very large, or small values for C_μ which cannot be allowed to become negative as this would produce a negative eddy viscosity which is not realistic. If C_μ becomes too large there are also numerical difficulties and hence its value was limited to the range $0 \leq C_\mu \leq 0.4$. The results for these two models are shown in Figs. 11-13.

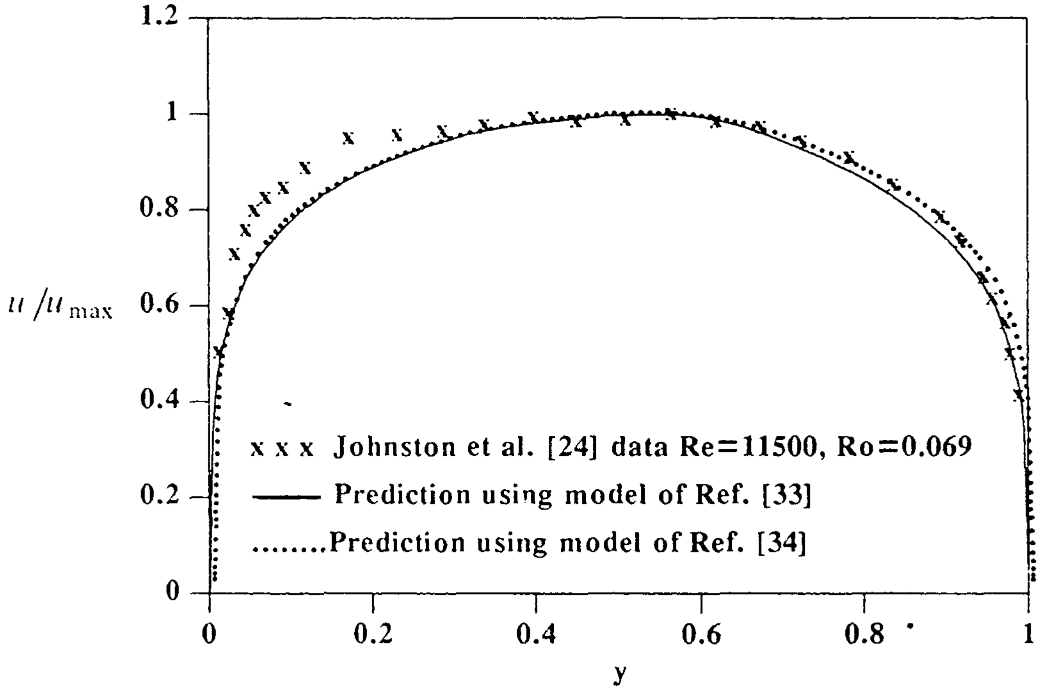

Fig. 11. Computed profiles with the modified C_μ models for Re=11500, Ro=0.069.

Neither model performs significantly better than the other nor much differently than the modified $k-\epsilon$ models tried earlier.

Algebraic Reynolds Stress Models

In an attempt to get better results two algebraic Reynolds stress models were tried. These models compute each of the individual Reynolds stress terms, τ_{ij}, with an algebraic equation so that the flow is no longer assumed to be isotropic. The first model investigated is that of Galmes and Lakshminarayana [32] whose equation for the stresses takes the form

$$\tau_{ij} = \frac{2}{3}k\delta_{ij} + k\frac{R_{ij}(1 - \frac{C_3}{2}) + (P_{ij} - \frac{2}{3}\delta_{ij}P)(1 - C_3)}{C_4 P} \tag{8}$$

where

$$P_{ij} = -\rho(\tau_{ik}\frac{\partial u_j}{\partial x_k} + \tau_{jk}\frac{\partial u_i}{\partial x_k})$$

$$R_{ij} = -2\rho\Omega_p(e_{ipk}\tau_{jk} + e_{jpk}\tau_{ik})$$

and P is production given by Eq. (7). This is a nonlinear set of algebraic equations for the stresses

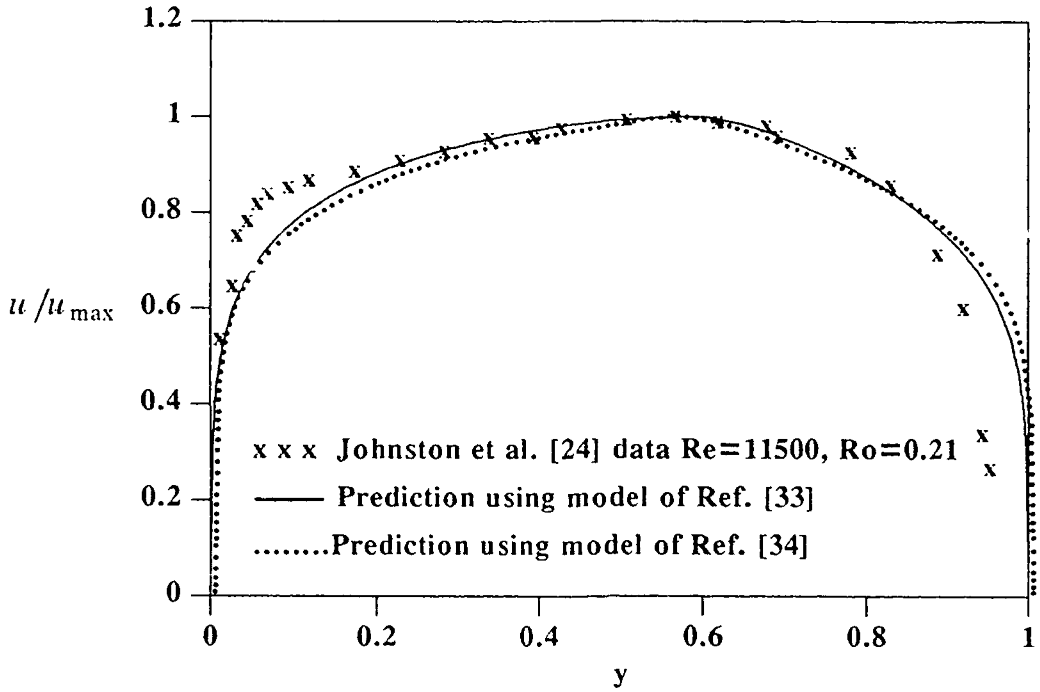

Fig. 12. Computed profiles with the modified C_μ models for Re=11500, Ro=0.21.

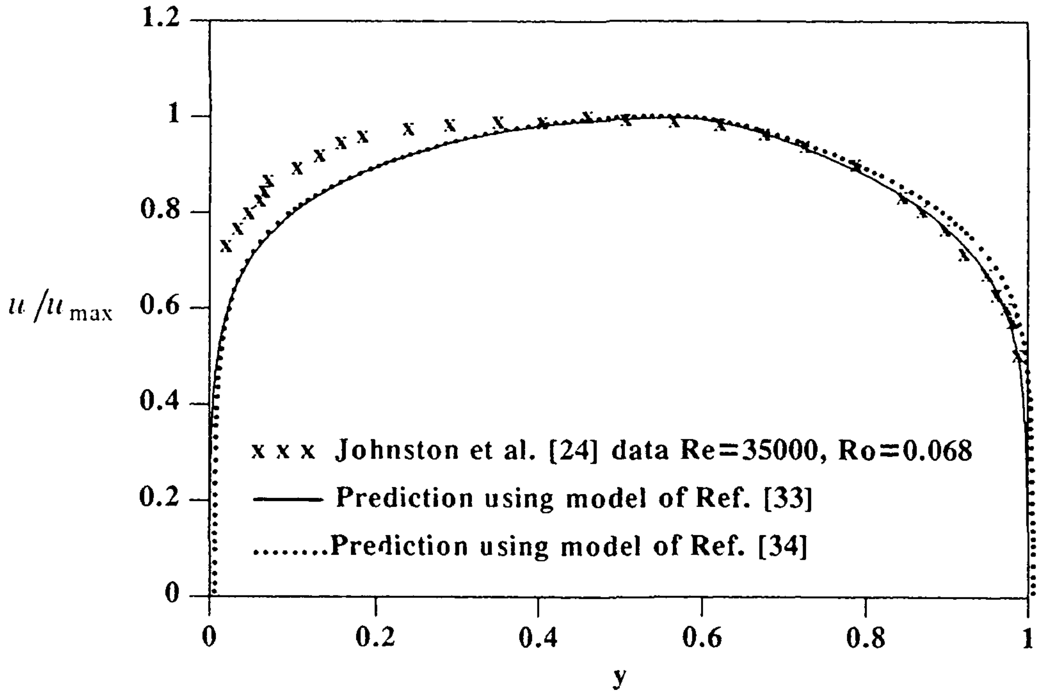

Fig. 13. Computed profiles with the modified C_μ models for Re=35000, Ro=0.068.

and can be difficult to solve for a complex flow field but reduces to a fairly simple form for the two-dimensional channel flow of interest. The previously investigated model of Pouagare and Lakshminarayana [33] is a simplification of this model and basically computes the shear stress term τ_{12} from it. The inclusion of the full algebraic model will differ from the model of Ref. [33] only by the inclusion of anisotropic effects in the normal stress terms. The results for the full model are shown in Figs. 14-16.

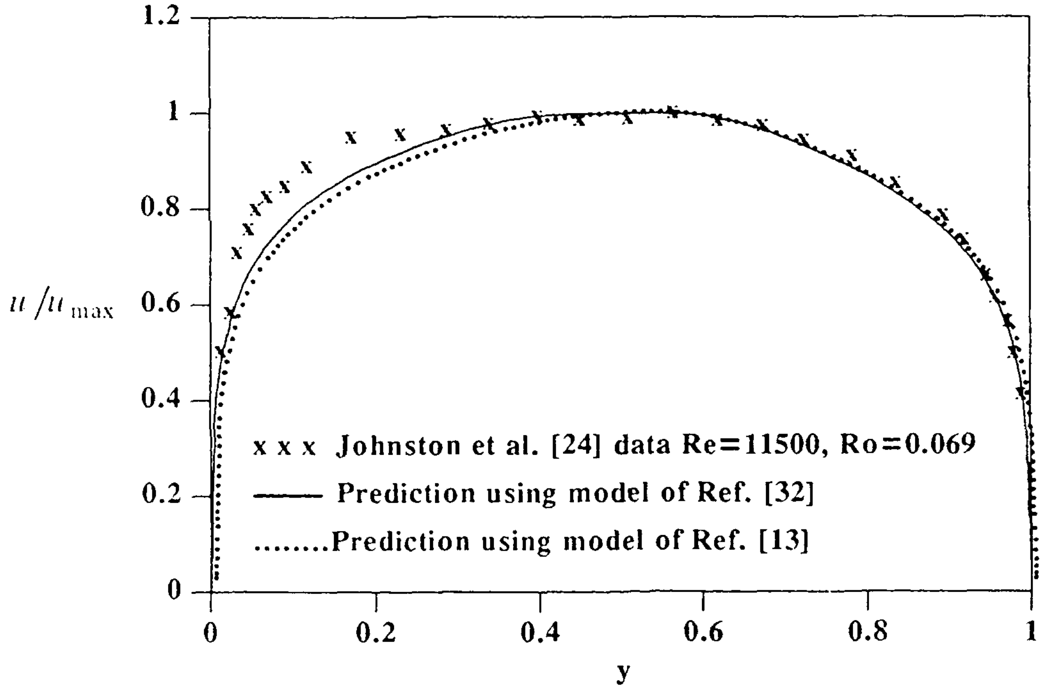

Fig. 14. Computed profiles using Algebraic Reynolds stress models for Re=11500, Ro=0.069.

They are nearly identical to the results computed using the model of Pouagare and Lakshminarayana [33] indicating that the normal stresses in the Navier-Stokes equations have little impact on the mean flow for this case.

In all of the above models, except that of Wilcox and Chambers [29], rotational effects depend on the Richardson number

$$R_i = \frac{-2\Omega(\frac{\partial u}{\partial y} - 2\Omega)}{(\frac{\partial u}{\partial y})^2}$$

Bardina et al. [35] showed that for rotating homogeneous shear flow the turbulent Reynolds stresses do not scale with the Richardson number indicating that these turbulence models may not be generally applicable to rotating shear flows. One model that does not depend on the Richardson number is that of Speziale [13] which is an extension of his earlier work [36]. This is another algebraic Reynolds stress model where the equations for the Reynolds stresses take the form

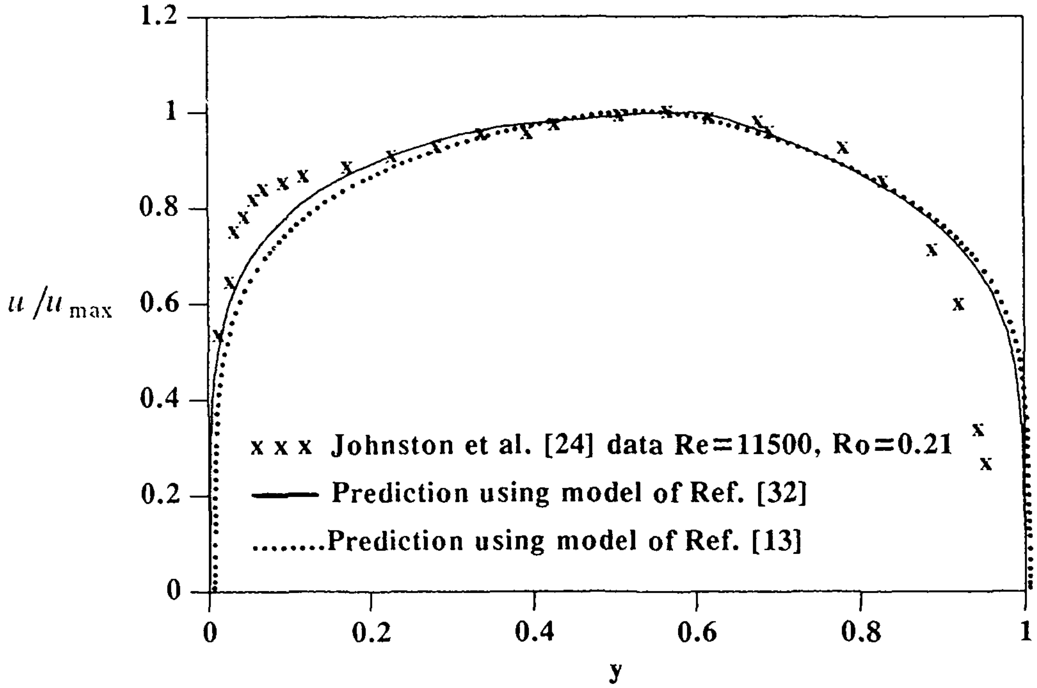

Fig. 15. Computed profiles using Algebraic Reynolds stress models for Re=11500, Ro=0.21.

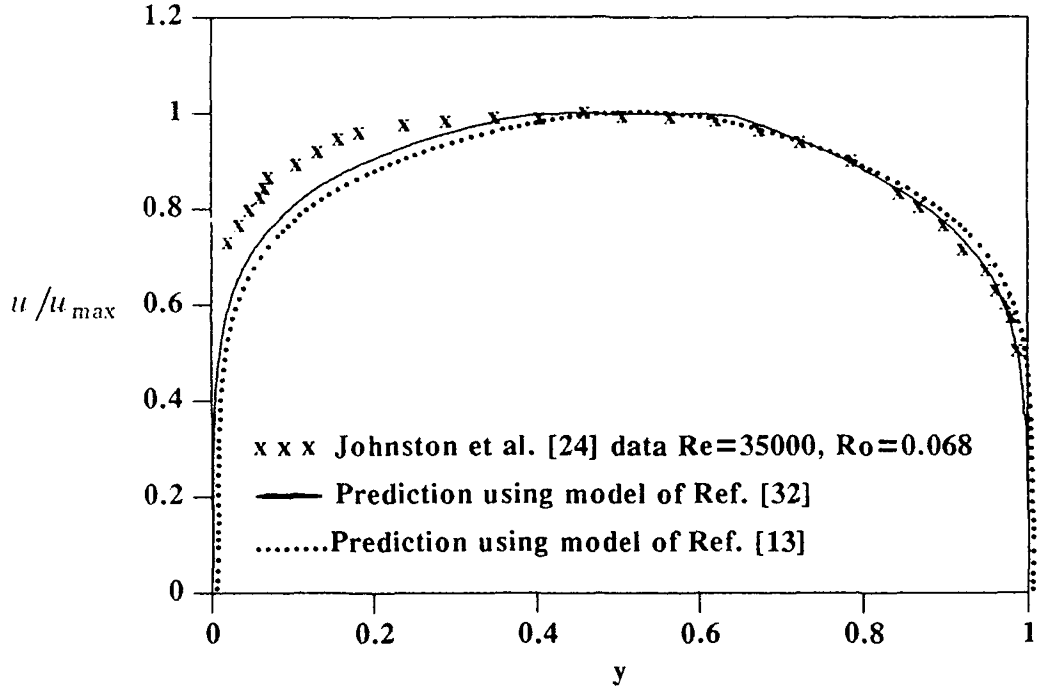

Fig. 16. Computed profiles using Algebraic Reynolds stress models for Re=35000, Ro=0.068.

$$\tau_{ij} = \frac{2}{3}k\delta_{ij} - 2C_\mu \frac{k^2}{\epsilon} S_{ij} + 4C_D C_\mu{}^2 \frac{k^3}{\epsilon^2}(\hat{S_{ij}} + S_{ik}S_{kj} - \frac{1}{3}S_{mn}S_{mn}\delta_{ij} + 2W_{ik}S_{kj} + 2W_{jk}S_{ki}) \tag{9}$$

where $C_D = 1.68$ and

$$\hat{S_{ij}} = \frac{\partial S_{ij}}{\partial t} + u_k \frac{\partial}{\partial x_k} S_{ij} - \omega_{ik}S_{kj} - \omega_{jk}S_{ki}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$W_{ij} = \omega_{ij} + e_{mji}\Omega_m$$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

However, when the present flow was computed with this model it produced a profile which was symmetric about the centerline like the standard $k-\epsilon$ model! To understand this we need to look at the contribution the rotation terms make to the Reynolds stresses. The normal stresses get modified by rotation in much the same way as that of the Galmes and Lakshminarayana [32] model with τ_{11} increased on the stable side and decreased on the unstable side. Conversely, τ_{22} gets increased on the unstable side and decreased on the stable side of the channel. The problem is that the shear stress dominates this flow as we saw earlier when the inclusion of normal stresses made little difference to the predictions. With the model of Speziale [13] the rotation has only a minor effect on the shear stress for the present flow and cancels out completely when the flow becomes fully developed.

To better include the effects of rotation in the model of Speziale [13] the current author made a simple modification based on the algebraic model of Ref. [32]. In the full Reynolds stress equations the shear stress is dependent on the normal stresses and this is modelled by Galmes and Lakshminarayana [32] in the form

$$\tau_{12} = C\Omega\frac{k}{\epsilon}(\tau_{11} - \tau_{22}) + \textit{additional terms}$$

This term was added to the definition of τ_{12} of Eq. (9) using $C = 0.25$. The results of this modification are shown in Figs. 14-16. As can be seen this modification does produce the desired asymmetry in the flow but the results are no better than those obtained with any of the other models and in fact are slightly worse.

CONCLUSIONS

The extension of the DTNS computer codes to compute flow in non-inertial reference frames is a rather straightforward process. However to find a turbulence model that can adequately compute the flow field is another matter. Standard algebraic eddy viscosity and two-equation $k-\epsilon$ models cannot properly account for the effects of rotation of the flow field. A variety of turbulence models have been investigated across the spectrum from simple modifications of the $k-\epsilon$ equations to somewhat complex algebraic relations for the individual Reynolds stresses. For the rather simple flow in a rotating channel the results were reasonable but all of the models underpredicted the velocity on the unstable side of the channel for all Rossby numbers studied.

The predictions did not compute the steep rise in the velocity evident in the experiment. For low Rossby numbers the predictions on the stable side of the channel are very good out past the symmetry plane. However, there are considerable differences between the predicted and experimental values near the wall for the high Rossby number case. Although it is difficult to determine which model is best, since all of the results are so similar, the models of Wilcox and Chambers [29] and Howard et al. [30] seem to slightly outperform the other more complex models. The above turbulence models should perform well enough for complex flows at relatively low Rossby numbers, such as a body in turn and some turbomachinery flows, but they would be quite questionable for high Rossby number cases. A more thorough comparison of these models and perhaps more complicated ones, needs to be done for more realistic flow fields before any firm conclusions can be made. Such an effort is beyond the scope of this work.

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